

# The Use of Burst Frequency Offsets in the Search for MH370

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## I. INTRODUCTION

ON 7 March 2014 at 16:41:43Z<sup>1</sup>, Malaysian Airlines flight MH370 departed Kuala Lumpur (KL) International Airport bound for Beijing. Less than an hour later, following the last recorded radio transmission from MH370 at 17:19:30Z, the plane's secondary radar transponder went offline. As evidenced by Malaysian military radar, the plane (registration number 9M-MRO) then veered off course unexpectedly, back-tracked across the Malaysian Peninsula, and was then tracked heading northwest from Penang through the Malacca Straits. After disappearing from radar at 18:22:12Z, it re-established a satellite communications (SATCOM) link with the Inmarsat satellite I-3F1 at 18:25:27Z. By analyzing a series of automated messages exchanged via that satellite between the plane and an Inmarsat ground station in Perth, Australia, it was determined that the plane continued to fly for six hours, before finally ceasing message exchange with the ground station at 00:19:37Z on 8 March 2014.

An initial analysis by Inmarsat suggested that MH370 had flown into the Southern Indian Ocean before SATCOM was ultimately lost. As summarized in [1] an intensive aerial and surface search was undertaken in the Southern Indian Ocean by an international search team during March and April 2014, with no MH370 related debris found. On 28 April 2014, the aerial search concluded and the search transitioned to an underwater phase [1]. The Australian Transport Safety Bureau (ATSB) took responsibility for the definition of the underwater search zone. It convened an international flight path prediction working group bringing together experts in satellite communications, statistical data processing and aviation, in order to estimate the most likely final location of flight MH370. The group consisted of representatives from the Australian Defence Science and Technology (DST) Group and the other organizations listed in the Acknowledgment section of this article.

New methods of analyzing the Inmarsat data were developed by the group, resulting in the release of reports concerning the likely final location of flight MH370 from the ATSB in August 2014 [2], October 2014 [3], and December 2015 [4]. Inmarsat also published an article regarding their contribution to the flight path reconstruction effort [5]. The DST Group contribution that assisted in the definition of an extended priority search area in December 2015 [4] has been detailed in [1]. This demonstrated how Bayesian analysis was

used to identify a high probability region of where the plane was believed to be at the time of last SATCOM transmission (00:19:37Z 8 March 2014). The DST Group Bayesian method used a prior probability distribution defined by the Malaysian military radar, a likelihood function describing the relationship between SATCOM measurements and the aircraft position and velocity during the flight, and a model of the aircraft dynamics.

SATCOM measurements used to define the likelihood function comprised burst timing offsets (BTOs), which give an indication of the range from the aircraft to the satellite, and burst frequency offsets (BFOs), which provide an estimate of the direction and velocity of the plane on the assumption of a given location. The BTO is well understood in the literature and was used to generate a series of arcs through which the plane must have crossed during its last six hours of flight (e.g. [1], [2], [5]). The BFO is a more complex measurement which is generally less well understood. In particular, the dependence of the BFO on and the sensitivity of the BFO to changes of vertical speed, ground track angle, and ground speed have not been covered in detail in the open literature. This article presents details on how the BFOs received from flight MH370 contributed to the definition of the underwater search area. Focus is given to the following:

- 1) A brief review of the statistical analysis of BFOs for several previous flights of 9M-MRO, which fed into the Bayesian model used for defining a priority underwater search area for the plane [1].
- 2) Details and examples of a method for presenting the difference between the measured BFO and the predicted BFO as a function of the aircraft's track angle for a given approximate location and an assumed ground speed. Summary results from this method were presented in [1] to show that if the plane was flying level at 18:39-18:41Z on 7 March, 2014, the BFOs from an unanswered ground-to-air telephone call show that the plane had to be tracking southwards by 18:39Z.
- 3) An analysis of the behavior of the frequency oscillator in the plane's satellite data unit (SDU) after power outage events such as those believed to have occurred twice during flight MH370, and how this affects the BFO.
- 4) How the results of the oscillator analysis, combined with direct Doppler analysis based on the vertical velocity of the aircraft, suggest that the final two BFOs recorded show that flight MH370 was rapidly descending and accelerating downwards at the time of the associated SATCOM transmissions. This analysis is consistent with

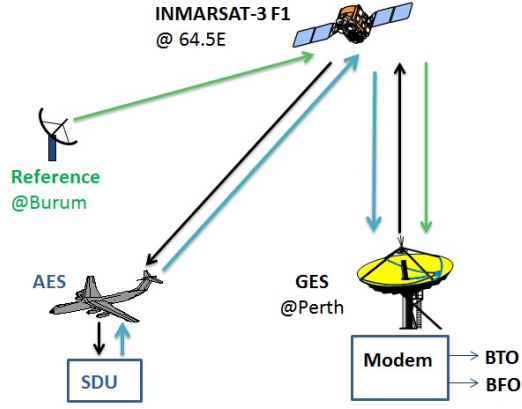


Fig. 1. System model of the satellite communication system [1].

the plane being in an uncontrolled descent at the time, as indicated in [6].

The remainder of this article is structured as follows. In Section II, a review of the SATCOM model is provided, along with a brief review of the BFO statistics. This serves as a summary of work previously presented in [1], for use in this article. Section III presents a method for representing the difference between the measured BFO and the predicted BFO as a function of the aircraft's track angle for a given location and an assumed ground speed. The settling behavior of the SDU's frequency oscillator for 9M-MRO after power-up is then detailed in Section IV. This is important because the SDU is believed to have undergone a power outage between 00:11Z and 00:19Z on 8 Mar 2014, immediately preceding the last two SATCOM transmissions from MH370. The effect of oscillator warm-up on the BFO after power-up can be used in bounding the descent rates for MH370 at 00:19:29Z and 00:19:37Z. In Section V an analysis of the descent rate of MH370 based on the last two BFO values is presented. This analysis derives lower and upper bounds on the descent rate at 00:19:29Z and 00:19:37Z. Conclusions are presented in Section VI.

## II. REVIEW OF SATCOM MODEL

The accident aircraft was fitted with a SATCOM terminal that used the Inmarsat Classic Aero system [5], which uses geosynchronous satellites to relay messages between aircraft and ground stations. During the flight, messages were passed between the aircraft and a ground receiving station located in Perth, Australia, via the Inmarsat-3F1 satellite. Figure 1 illustrates the SATCOM system in use during the flight. The aircraft is referred to as the Aircraft Earth Station (AES) and the ground receiving unit is referred to as the Ground Earth Station (GES). Inmarsat-3F1 is a satellite in geosynchronous orbit at 64.5° East longitude and it was used exclusively for the duration of the flight.

An AES is equipped with an SDU comprising a satellite modem with auxiliary hardware and software. Transmission of data over the satellite is via bursts which are scheduled to arrive at the GES at a specified time and with a given frequency. As explained in [1], [5], communications from multiple users are coordinated by the allocation of different

time and frequency slots to each user. This is done without knowledge of individual AES locations or precise knowledge of the satellite location. Therefore, messages from a given AES might not arrive at the GES at exactly the expected time, and generally would arrive slightly later. The difference between the expected time of arrival (based on a nominal assumed position for the satellite and the AES) and the actual time of arrival is referred to as the BTO. The BTO is a measure of how far the aircraft is from the sub-satellite position<sup>2</sup>.

The relative velocity between the satellite and the AES, as well as between the satellite and the GES, leads to a Doppler frequency offset on the signals received at the GES. Coupled with small frequency offsets inherent in the reference frequency oscillators in the AES, satellite and GES, this results in a net difference between the expected and actual frequency of the signal presented to the modem in the GES for a given user. Frequency compensations applied onboard the aircraft (aircraft induced Doppler pre-compensation) and at the ground station (Enhanced Automatic Frequency Correction, which utilizes the reference signal transmitted from a reference station in Burum, Netherlands), [1], [5] serve to reduce the possible difference between the expected and actual frequency of the messages received from the aircraft. The residual difference between the expected frequency of each communications burst and the actual received frequency is referred to as the BFO. The BFO is a function of the relative velocity between the aircraft and the satellite. Given that the satellite position and velocity are accurately known, the BFO provides information about the aircraft velocity vector.

### A. Review of BFO Statistics

Based on 20 previous flights of 9M-MRO in the week leading up to the accident flight (see [1] for further details), a histogram was produced for the difference between the predicted BFO (based on known details of the plane and the satellite's position and velocity) and the measured BFO (based on Inmarsat ground station logs). This difference (i.e. predicted minus measured) is referred to as the BFO error. The histogram of the BFO error is shown in Fig. 2, along with a Gaussian distribution fit line. It can be seen that the distribution is somewhat Gaussian. The standard deviation of the BFO error was found in [1] to be 4.3 Hz. Whilst it is generally reasonable to apply bounds on the possible BFO error based on  $\pm 3$  standard deviations as was done for the approximate analysis described in [6], for the purpose of the descent analysis presented later in Sec. V, it is assumed the BFO error is strictly bounded on the larger interval  $[-28, +18]$  Hz, which corresponds to the bounds of all 2501 observed valid<sup>3</sup> in-flight BFO error values available from the preceding 20 flights of 9M-MRO.

## III. EFFECTS OF POSITION AND VELOCITY ON THE BURST FREQUENCY OFFSET

In [1], the BFO is defined mathematically at time step  $k$  as the sum of a noiseless component  $h_k^{\text{BFO}}$  and a scalar  $w_k^{\text{BFO}}$

<sup>2</sup>The sub-satellite position is the point on the earth directly below the satellite.

<sup>3</sup>One outlier was removed as explained in Sec. IV.

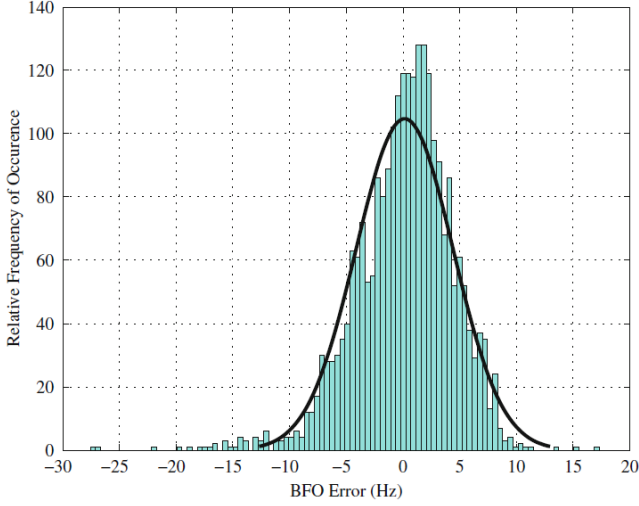


Fig. 2. Histogram of BFO errors for 20 flights of 9M-MRO prior to MH370 (reproduced from Fig. 5.5 of [1]).

that represents the BFO noise. The noiseless component of the BFO is defined in [1] as the sum:

$$h_k^{\text{BFO}}(\mathbf{x}_k, \mathbf{s}_k) = \Delta F_k^{\text{up}}(\mathbf{x}_k, \mathbf{s}_k) + \Delta F_k^{\text{down}}(\mathbf{s}_k) + \delta f_k^{\text{comp}}(\mathbf{x}_k) + \delta f_k^{\text{sat}}(\mathbf{s}_k) + \delta f_k^{\text{AFC}}(\mathbf{s}_k) + \delta f_k^{\text{bias}}(\mathbf{x}_k, \mathbf{s}_k), \quad (1)$$

where

- $\mathbf{x}_k$  denotes the state vector of the aircraft;
- $\mathbf{s}_k$  denotes the state vector of the satellite;
- $\Delta F_k^{\text{up}}(\mathbf{x}_k, \mathbf{s}_k)$  is the uplink (aircraft to satellite) Doppler shift;
- $\Delta F_k^{\text{down}}(\mathbf{s}_k)$  is the downlink (satellite to ground station) Doppler shift;
- $\delta f_k^{\text{comp}}(\mathbf{x}_k)$  is the frequency compensation applied by the aircraft;
- $\delta f_k^{\text{sat}}(\mathbf{s}_k)$  is the variation in satellite translation frequency;
- $\delta f_k^{\text{AFC}}(\mathbf{s}_k)$  is the frequency compensation applied by the ground station receive chain;
- $\delta f_k^{\text{bias}}(\mathbf{x}_k, \mathbf{s}_k)$  is a slowly varying bias due to errors in the aircraft and satellite oscillators and processing in the SDU.

By treating the bias  $\delta f_k^{\text{bias}}(\mathbf{x}_k, \mathbf{s}_k)$  as a constant determined at the source tarmac for any particular flight, as was done in [5] for MH370, the time-varying component of the bias during a particular flight can be considered as part of the BFO noise (indeed this was done when compiling the results used to obtain the BFO error histogram shown in Fig. 2). Details regarding the terms  $\delta f_k^{\text{sat}}(\mathbf{s}_k)$  and  $\delta f_k^{\text{AFC}}(\mathbf{s}_k)$  are provided in [5]. Tabulated values of the sum of these two terms were provided by Inmarsat to the MH370 Flight Path Reconstruction Group to use in estimating the likely trajectory flown. These two terms depend on the satellite state  $\mathbf{s}_k$  only, and not on the aircraft state  $\mathbf{x}_k$ . Moreover, the downlink Doppler  $\Delta F_k^{\text{down}}(\mathbf{s}_k)$  does not depend on the location or velocity of the aircraft, and can be calculated given the aircraft's uplink

frequency of the SATCOM messages and the known satellite state at any given time.

Having treated the slowly time-varying bias  $\delta f_k^{\text{bias}}(\mathbf{x}_k, \mathbf{s}_k)$  as a constant determined at the source tarmac and being able to calculate or read from tables the other terms, equation (1) can be re-written as:

$$h_k^{\text{BFO}}(\mathbf{x}_k, \mathbf{s}_k) = \Delta F_k^{\text{up}}(\mathbf{x}_k, \mathbf{s}_k) + \delta f_k^{\text{comp}}(\mathbf{x}_k) + \delta f_k^{\text{det}}(\mathbf{s}_k), \quad (2)$$

where  $\delta f_k^{\text{det}}(\mathbf{s}_k)$  is effectively a known deterministic value for any time step  $k$ . The other terms in (2) couple the aircraft state  $\mathbf{x}_k$  by way of the aircraft position and velocity to the BFO as per the following equations adapted from [1]<sup>4</sup>:

$$\Delta F_k^{\text{up}}(\mathbf{x}_k, \mathbf{s}_k) = \frac{F^{\text{up}}}{c} \frac{(\mathbf{v}_s - \mathbf{v}_x)^T (\mathbf{p}_x - \mathbf{p}_s)}{|\mathbf{p}_x - \mathbf{p}_s|}, \quad (3)$$

$$\delta f_k^{\text{comp}}(\mathbf{x}_k) = \frac{F^{\text{up}}}{c} \frac{(\hat{\mathbf{v}}_x)^T (\hat{\mathbf{p}}_x - \hat{\mathbf{p}}_s)}{|\hat{\mathbf{p}}_x - \hat{\mathbf{p}}_s|}, \quad (4)$$

where the dependence on the time step  $k$  on the right hand side of the equations has been removed for simplicity of notation. In equations (3) and (4),  $|\cdot|$  is the three dimensional Cartesian distance, and:

- $F^{\text{up}}$  is the uplink carrier frequency;
- $c$  is the speed of light;
- $\mathbf{v}_s$  is the velocity vector of the satellite;
- $\mathbf{v}_x$  is the velocity vector of the plane;
- $\mathbf{p}_s$  is the position vector of the satellite;
- $\mathbf{p}_x$  is the position vector of the plane;
- $\hat{\mathbf{v}}_x$  is the SDU's estimate of the plane's velocity vector, which is obtained using the plane's track angle and ground speed, whilst assuming the vertical speed is zero;
- $\hat{\mathbf{p}}_s$  is the SDU's estimate of the position vector of the satellite, which assumes the satellite is at its nominal orbital slot of 0 degrees North and 64.5 degrees East;
- $\hat{\mathbf{p}}_x$  is the SDU's estimate of the position vector of the plane, which is obtained using the plane's latitude and longitude, whilst assuming the plane is at sea level.

The SDU's estimates of the satellite and aircraft's positions, along with its estimate of the aircraft's velocity, are used to perform a pre-compensation of the aircraft's component of the uplink Doppler. This compensation is defined in (4). As the satellite moves closer to its nominal position (i.e. the position the SDU assumes the satellite is at), it makes sense that the SDU's estimate of the aircraft's uplink Doppler contribution will become closer to the aircraft's true uplink Doppler contribution. Accordingly, considering (2) to (4), the BFO depends less on the aircraft's ground velocity vector (i.e. its ground speed and track angle). However, when the satellite's true location is further from its nominal position (see Fig. 3), the BFO can be used to make inferences about the ground velocity of the plane. DST Group created a model based on equations (2) to (4) allowing the BFO to be predicted as a function of hypothesized ground speeds and track angles, given an assumed location of the plane at a given point in time. Following on from the preceding discussion, it is seen

<sup>4</sup>Note that the sign convention used in [1] is opposite to that used in (3)

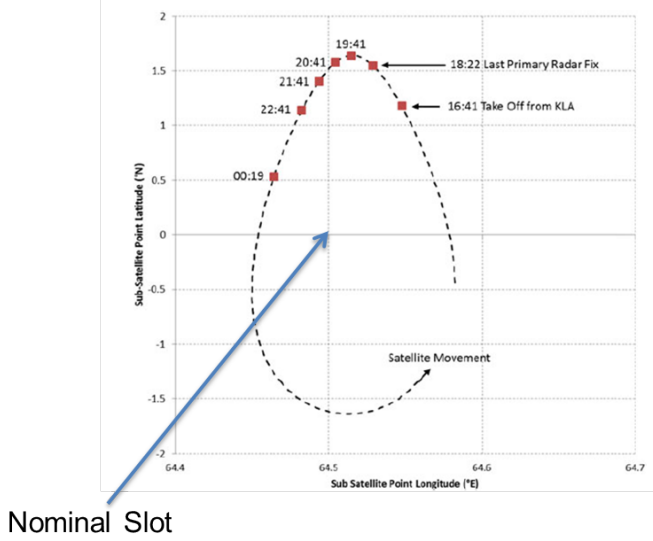


Fig. 3. Sub-satellite Track During the MH370 Flight. (Adapted from ([5], Fig. 8), with permission). Note that the longitude scale is about 10 times finer than the latitude scale, causing the sub-satellite track to appear more stretched out in latitude than it really is.

that the sensitivity of the BFO to the ground track angle of the plane for a given speed will be greater when the satellite is further from its nominal position. Viewing a set of curves of BFO error vs track angle for an example ground speed will demonstrate this. It is first beneficial to consider the sub-satellite position ground track for the duration of flight. The sub-satellite position is shown for the period of the flight in Fig. 3. The sub-satellite position moves in an anti-clockwise direction over the time period shown. Specific satellite positions in latitude and longitude at various times throughout the flight are shown in the figure.

Fig. 4 shows how the BFO error varies for a range of postulated ground track angles at times corresponding to the familiar BTO rings published in [2], [5], and reproduced here in Fig. 5 for convenience. The assumed positions necessary for the generation of the BFO error curves in Fig. 4 are shown in Table I. It should be noted that the points shown in the table are examples only. Whilst all but the “19:41Z (alternate)” position are chosen to lie on or very close to the BTO rings in Fig. 5, and correspond to a generally southward track angle after 18:39Z, they do not necessarily correspond to the most likely paths shown in [1]. This is reasonable since the aim of Fig. 4 is to demonstrate that the magnitude of the peak-to-peak variation of BFO error (summarized in Table II) is dominated by the difference between the sub-satellite position and the nominal sub-satellite position (see right-most column of Table II). The effect of the assumed AES position on peak-to-peak variation of BFO error for a full range of possible track angles is seen to be secondary in comparison. This is demonstrated by the BFO error curve presented in Fig. 4 for an alternate postulated position of the AES at 19:41z (denoted “1941<sub>alt</sub>”), which was chosen not to lie on the 19:41Z ring, but rather much closer to the nominal satellite position (see Table I). Despite the large difference in assumed AES location,

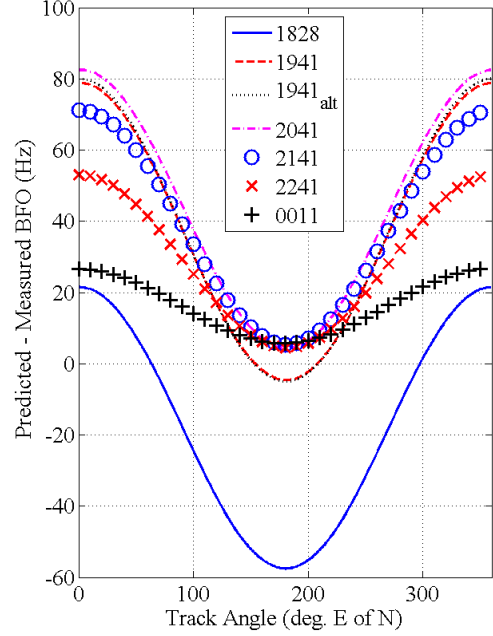


Fig. 4. BFO Errors as a Function of Track Angle During the MH370 Flight (Assumed ground-speed is 450 kts.)

TABLE I  
EXAMPLE POSITIONS FOR MH370 AT ARC TIMES

Timestamp	Latitude (Degrees)	Longitude (Degrees)
18:28Z	6.54 N	95.69 E
19:41Z	1.96 S	93.37 E
19:41Z (alternate)	0 N	80 E
20:41Z	10.93 S	91.65 E
21:41Z	19.15 S	89.99 E
22:41Z	27.25 S	88.25 E
00:11Z	38.67 S	85.11 E

this curve is almost identical to the “1941” curve, and the peak-to-peak variation shown in Table II is only different by 2 Hz. On the other hand, consider the last row of Table II. This corresponds to the time 00:11Z, at which time the sub-satellite distance from the nominal sub-satellite position is less than half of its 19:41Z value. In this case, for the assumed ground speed of 450 kts, the peak-to-peak difference in predicted BFO for a North or South track is only 21 Hz, which is only around  $\pm 2.5$  times the standard deviation of the BFO noise. It should be noted that although the BFO effectively indicates less about the plane’s ground track angle and speed in the latter portion of the flight, the DST Group Bayesian method [1] implicitly takes this into account. This is consistent with the fact that the latitudinal spread of the probability density function of 7<sup>th</sup> arc crossing points in the Southern hemisphere in ([1], Fig. 10.3) is similar irrespective of whether or not the BFO is used.

Fig. 4 also shows that for even the southern-most track angles, at and after 20:41Z, there would have been a positive BFO error (for the assumed ground speed of 450 kts). When comparing specific track angles for the various curves with



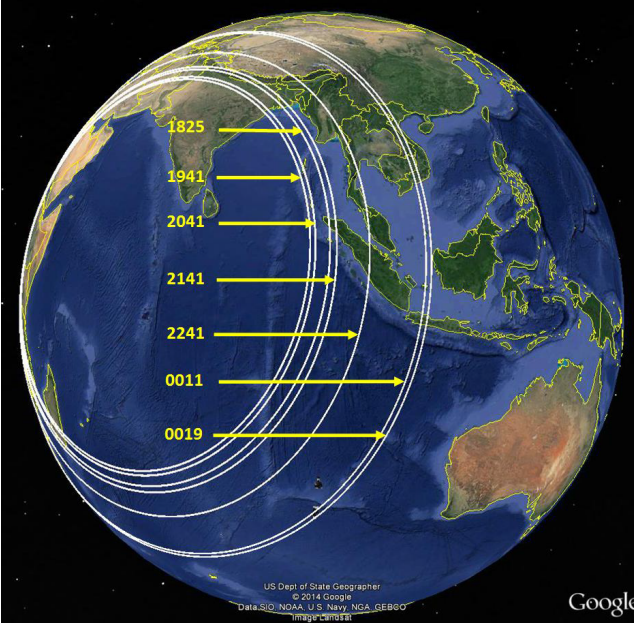


Fig. 5. BTO Rings During the MH370 Flight [2]. Note that latitude of the arrows shown for each ring is arbitrary.

TABLE II  
PEAK TO PEAK PREDICTED BFO VARIATION DURING THE MH370 FLIGHT

Timestamp	Peak-to-Peak BFO Variation (Hz)	Sat. Offset (km)
18:28Z	79	1229
19:41Z	83	1277
19:41Z (alt.)	85	1277
20:41Z	78	1233
21:41Z	66	1115
22:41Z	49	935
00:11Z	21	607

the posterior control angles (which were also ground track angles) in ([1], Fig. 10.5), it is seen that the residual BFO measurement errors shown in ([1], Fig. 10.7) are similar to the translation in BFO that would need to be applied to Fig. 4 to have zero BFO error<sup>5</sup>.

#### A. Effect of uncompensated vertical velocity

The results just presented show that the BFO is a less effective discriminator of the plane's ground track angle as the satellite moves closer to its nominal orbital location, as occurred from 19:41Z until the loss of SATCOM at 00:19:37Z. The vertical speed of the plane is not used in the SDU Doppler compensation. As such, regardless of the true satellite location with respect to its nominal location, there is a direct contribution of Doppler due to the proportion of the vertical velocity vector projected onto the radial direction to the satellite at any given time. It is straightforward to understand that if the plane

were directly below the satellite, the vertical velocity vector would be fully towards or away from the satellite (i.e. no projection is required in that case) depending on whether the plane were climbing or descending, respectively. The direct contribution of the Doppler to the BFO in that case would be governed by the following standard Doppler equation.

$$\Delta F_k^{\text{up}}(\mathbf{x}_k, \mathbf{s}_k) = \frac{v_z \cdot F^{\text{up}}}{c}, \quad (5)$$

where  $F^{\text{up}}$  and  $c$  are as previously defined, and  $v_z$  is the vertical speed of the plane. Substituting an uplink frequency of 1646.6525 MHz (the uplink frequency stated in [5]) and a vertical velocity of 100 feet per minute (fpm), equivalent to 0.508 meters per second, equation (5) implies that the predicted BFO would increase by 2.8 Hz per 100 fpm of climb rate or decrease by 2.8 Hz per 100 fpm of descent rate if the plane were directly below the satellite. This is the maximum possible contribution of the plane's climb or descent rate to the BFO. In the more general case, equation (5) is moderated by the sine of the elevation angle  $\theta$  to the satellite in order to project the plane's climb or descent rate onto the radial direction to the satellite. This is expressed as

$$\Delta F_k^{\text{up}}(\mathbf{x}_k, \mathbf{s}_k) = \frac{v_z \cdot F^{\text{up}} \sin(\theta)}{c}. \quad (6)$$

As such, at the so-called 7<sup>th</sup> arc, where the elevation to the satellite is 38.8 degrees, the contribution to the BFO of climb or descent rate is reduced to approximately +1.7 Hz or -1.7 Hz per 100 fpm respectively.

#### B. BFO Trend During the MH370 Flight

The measured BFOs from 19:41Z to 00:11Z are shown in Fig. 6. A line-of-best fit for the interval 19:41Z to 00:11Z is also plotted. This line is extended forward to the time of the 00:19Z log-on resulting in an expected BFO of roughly 254 Hz<sup>6</sup>. Considering again the track angle curves earlier presented in Fig. 4, the 00:11Z BFO error value for the southern-most track is roughly 6 Hz, meaning the measured BFO of 252 Hz was 6 Hz lower than the lowest value it could have been (assuming roughly level flight at approximately 450 kts). If we assume the same error value at 00:19Z, and use the extrapolated BFO, assuming a south track with level flight and similar ground speed, the expected BFO value is 260 Hz. The difference between this expected value and the BFO values logged at 00:19:29Z and 00:19:37Z will be used in Sec. V to derive bounds on the possible descent rates of MH370 at the times of the last two SATCOM transmissions.

The linear trend in Fig. 6 is also extended back by 1 hour to the time of the first unanswered satellite telephone call to MH370. The mean of the BFOs logged during that call attempt is also shown. Since the mean of the BFOs from the 18:39-18:41Z call attempt are in broad agreement with the linear trend observed in the BFOs from 19:41Z to 00:11Z (for which the BTOs themselves were consistent with straight and level

<sup>5</sup>Note that whilst in Fig. 4, the predicted minus measured BFO is shown, the residual BFO errors presented in ([1], Fig. 10.7) were measured minus posterior BFO, hence there is a necessary sign inversion when comparing the two sets of results.

<sup>6</sup>Note the BFOs observed at 00:19Z were much lower than the expected value, which as shown later in this article provides evidence of the descent of MH370 at that time.

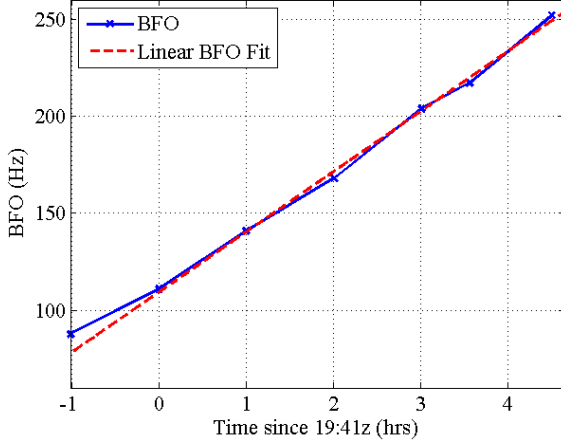


Fig. 6. Measured BFOs During the MH370 Flight.

flight [1]), this supports the finding in [1] that there were most-likely no major turns after the unanswered call attempt (see ([1], Fig. 10.5)).

#### IV. EFFECT OF SDU STARTUP ON THE BFO

At 18:25:27Z 7 March 2014, the Inmarsat GES in Perth received a SATCOM log-on request from 9M-MRO. A series of messages were exchanged in the following few minutes as part of a standard log-on sequence. The BFOs over those minutes displayed somewhat unusual behavior in that (barring the first BFO) the BFO appeared to be exhibiting a transient settling behavior. This is shown in Fig. 7. It was noted in [5] that the spike in BFO observed at this time<sup>7</sup> was not fully understood and whilst originally attributed to a possible aircraft maneuver it could also be related to actual frequency changes occurring during the logon sequence. A subsequent study [7] by the SATCOM sub-group of the MH370 Flight Path Reconstruction group revealed this was most likely due to power-on frequency drift and subsequent stabilization of the oven-controlled crystal oscillator (OCXO) in the SDU. In [7], a number of different SDUs were tested to investigate the effects of power outages on the BFO during SDU power on, triggering a SATCOM logon such as that which occurred for MH370 at 18:25:27Z. It was found that whilst the frequency settling behavior was different for each individual SDU, any given SDU displayed repeatable behavior for a fixed outage duration, and similar settling characteristics for outages of different duration.

Based on sequences of Inmarsat BFO logs for 9M-MRO concerning four log-ons after periods of SATCOM outage in the week leading up to the accident flight, it was found (in [7]) that the SDU in 9M-MRO resulted in a BFO that was initially too high at the time of log-on, followed by a simple decay over the next few minutes to a steady-state value. Regarding the log-on event at 18:25:27Z for MH370, it was noted that there was a non-zero bit error rate (BER) associated with the log-on request at that time. The associated received signal level

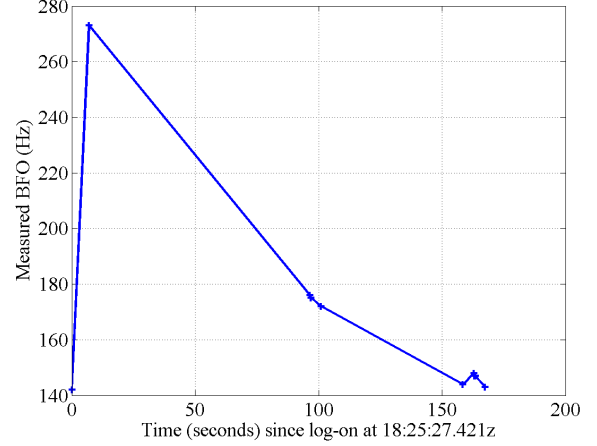


Fig. 7. Measured BFOs following the 18:25:27Z log-on message from MH370.

and carrier-to-noise density ratio ( $C/N_0$ ) were also unusually low. As such, the first BFO was deemed untrustworthy. Note that this is not the only time such a behavior was observed for the SDU in 9M-MRO. In the analysis of BFOs presented in [1], there was an outlier in the BFO (with an error of -170 Hz) found to have occurred for a SATCOM message associated with 9M-MRO in one of the 20 previous flights of 9M-MRO. This was discarded as an outlier and not presented in [1]. It too had a non-zero BER and a low  $C/N_0$  (37.6 dBHz compared to surrounding values between 41.5 and 42 dBHz). With the removal of the untrustworthy BFO logged for the first message in the sequence shown in Fig. 7, the 18:25Z log-on from MH370 was also determined to follow the simple decay trend observed in other instances.

It was noted by DST Group that if the simple BFO decay trend established after SDU power-up found in [7] for five cases with the 9M-MRO SDU also occurred during the MH370 SATCOM log-on event beginning at 00:19:29Z on 8 March 2014, then it would allow bounds to be established on the BFO that would have been expected for MH370 if the plane was flying level at that time. This in turn would allow the determination of bounds on the possible descent rates of MH370 during that final log-on event. In order to build confidence that the simple BFO decay trend would hold for the 00:19Z log-on event, DST Group reviewed additional Inmarsat logs for 9M-MRO, corresponding to the period 22 February to 28 February 2014. Two additional cases were identified in which 9M-MRO logged back onto an Inmarsat satellite after a sustained period of SATCOM outage. The sequence of BFOs observed in these two cases, along with the five already considered by the SATCOM working group in [7], are shown in Fig. 8, and details about each of the log-ons are given in Table III.

The periods of SATCOM outages followed by log-on events were identified from Inmarsat-provided ground-station logs by identifying sequences of three or more unsuccessful log-on interrogations to 9M-MRO (suggesting the SDU was likely powered off) followed by a log-on to the satellite system initiated from 9M-MRO at some later time. The exact duration of

<sup>7</sup>When viewed on a time-scale of hours as shown in Fig. 9 of [5], this behavior looks like a spike in BFO between 18:25Z and 18:28Z.

TABLE III  
DETAILS OF LOG-ON SEQUENCES USED FOR ANALYSIS

Identifier	Date and Timestamp of Log-on	Duration of Preceding Power Outage	Any Other Comments
Log-on 1	23rd Feb. 23:57Z	< 442 and probably > 381 minutes	After scheduled A1 maintenance check, some non-zero BERs
Log-on 2	26th Feb. 14:11Z	Between 295 and 354 minutes	
Log-on 3	5th Mar. 03:06Z	Between 35 and 95 minutes	
Log-on 4	6th Mar. 13:29Z	Between 43 and 103 minutes	
Log-on 5	6th Mar. 15:02Z	Between 35 and 92 minutes	
Log-on 6	7th Mar. 12:50Z	Between 228 and 288 minutes	Some non-zero BERs
Log-on 7	7th Mar. 18:25Z	Between 20 and 78 minutes	First point untrustworthy

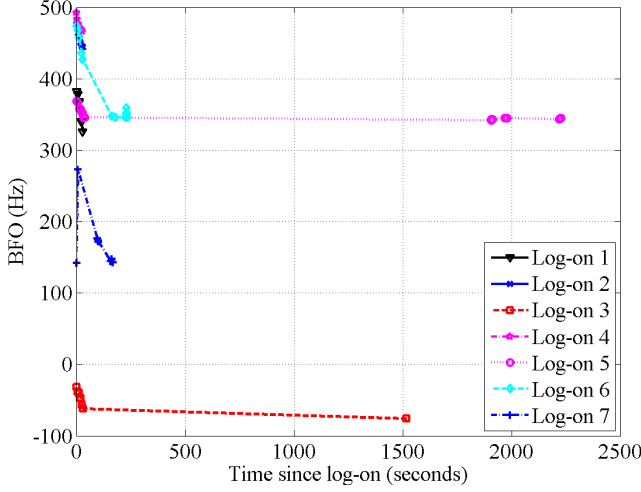


Fig. 8. Measured BFOs for 7 log-ons of 9M-MRO.

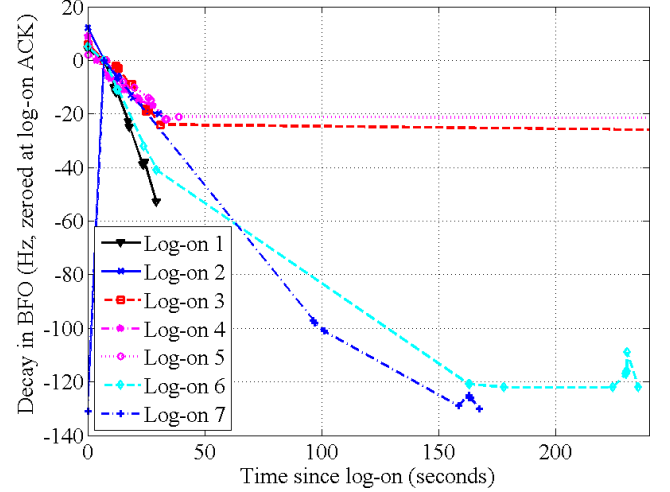


Fig. 9. Decay in measured BFOs for 7 log-ons of 9M-MRO.

the power-off period for the SDU was unable to be determined from the logs, however the timestamps associated with the unsuccessful log-on interrogations and subsequent 9M-MRO initiated log-ons were used to determine bounds on the outage time as shown in the 3<sup>rd</sup> column of Table III. Additional comments about each log-on sequence are given in the final column. Whilst some of the BERs for log-ons 1 and 6 were non-zero, the BFOs did not appear to be outliers, and were therefore considered valid. Note that given log-on 1 occurred immediately after the A1 maintenance check conducted for 9M-MRO at Kuala Lumpur International Airport (KUL) on 23 February 2014, it is possible that part of the SATCOM outage could be attributed to the plane being in a hangar rather than the SDU power outage. The uncertainty window in the length of the outage duration for this and all other log-ons observed is roughly 60 minutes; this relates to timers in the Inmarsat satellite ground stations that result in a log-on interrogation being sent to the plane after roughly 60 minutes of SATCOM inactivity. Indeed, it is this same timing responsible for the roughly 60 minute separation between the four BTO arcs from 19:41Z to 22:41Z (e.g. [2], [5]). It is also of note that based on other evidence related to MH370 (e.g. secondary radar transponder loss of signal at 17:22Z) that SDU power was probably first lost at 17:22Z, in which case the SDU power outage duration for log-on 7 would be approximately 63 minutes.

For log-ons 3 and 5, there were some points available about half an hour after the log-on in each case (noting the plane was on the tarmac for the whole time) that demonstrate the BFO appears to have fully settled to a steady-state value. Log-ons 1 to 5 only lasted for less than 1 minute after the log-on request message. The results for log-ons 6 and 7, which had log-on sequences lasting a few minutes suggest a settling behavior of approximately 3 minutes duration. For all observed log-on sequences, there seems to be a simple decay of BFO over a few minutes after SDU startup and log-on request (see also Fig. 9). It is therefore likely a similar decay was occurring during the SDU log-on sequence beginning at 00:19:29Z on 8 March 2014 for MH370.

In order to establish what the likely range of steady-state equivalent BFOs would have been at 00:19Z for MH370 (and therefore infer a likely range of possible descent rates), it is useful to re-plot the curves from Fig. 8 translated vertically such that the log-on acknowledgment has 0 Hz offset. This is done in Fig. 9, which has also been zoomed in the time-axis for clarity.

Recall that the two messages at 00:19:29Z and 00:19:37Z are a log-on and log-on acknowledgment respectively. It can be seen from Fig. 9 that the log-on acknowledgment BFOs are in the range of [0,12] Hz lower than the log-on BFOs. It can also be seen that the maximum difference between the log-on BFO and the settled BFO value is in the range [33,142] Hz. The only

log-on that doesn't appear to be approaching a settled value in this range is log-on 1, for which it is anticipated that if more data points were available several minutes later, as was the case for log-ons 3, 5, 6 and 7, the settling behavior would be similar to that for log-ons 6 and 7, bearing in mind the outage duration for log-on 1 is probably at least 381 minutes, and the BFO already dropped substantially in the first 30 seconds after log-on.

## V. BOUNDING THE DESCENT RATES OF MH370

The results from Sections III and IV can be combined in order to provide bounds on the descent rate of MH370 implied by the BFOs from the two last SATCOM messages for the flight, which occurred on 8 March 2014 at 00:19:29Z and 00:19:37Z. In this section, it is shown how this is done for two different possibilities that could explain the attempted SATCOM log-on from 9M-MRO at 00:19Z. In [2] the most likely cause of this log-on was stated to be a power interruption resulting from insufficient fuel and subsequent engine flameout. It could also have been due to a temporary software failure, a loss of systems providing critical input to the SDU, or a loss of the SATCOM link due to aircraft attitude being such that the line-of-sight to the satellite is blocked. If it was indeed a power interruption to the SDU caused by loss of fuel and subsequent reboot using the Auxiliary Power Unit (APU), the SDU would be without power for about 4-6 minutes. In this case, the results of Sec. IV need to be considered when interpreting the last two BFOs. If on the other hand, the power loss was momentary (resulting in a reset of the SDU) or if the temporary SATCOM outage leading to the log-on request was due to one of the other listed reasons, there would be no "warm-up drift" to consider, so the results of Sec. IV would not need to be applied. Both cases are considered separately in the following two sub-sections, and then combined overall bounds are presented.

### A. Hypothesis 1: SATCOM outage due to insufficient fuel

In the event that the SDU log-on at 00:19:29Z was due to engine flame-out, followed by a restart of the SDU using power from the APU (as was stated to be the most likely scenario in [2]), the SDU outage preceding the log-on would have lasted about 4-6 minutes. This would result in cooling of the OCXO in the SDU. During the start-up sequence of the SDU, the oven warms the crystal to a required temperature, and an oven-ready signal is asserted leading to the SDU issuing a log-on request [7]. Subsequent to this, final stabilization of the OCXO for the case of the SDU in 9M-MRO has been found in all 7 considered cases to result in a simple BFO transient decay over the first few minutes after log-on. The difference between log-on BFO and the settled BFO values was noted in Sec. IV to be between 33 and 142 Hz. Since other testing by the SATCOM Working Group [7] revealed a given SDU always displays a similar BFO settling behavior after start-up (see Sec. IV), it is reasonable to assume a similar settling behavior was occurring for MH370 at 00:19:29Z and 00:19:37Z. As such, the recorded BFO at 00:19:29Z was most likely between 33 and 142 Hz higher than it would have

been if there was no start-up drift present. Also noting from Sec. IV that the BFOs for the log-on acknowledge messages (2nd message in each sequence) were between 0 and 12 Hz lower than those for the log-on request<sup>8</sup>, it can be inferred that the recorded BFO at 00:19:37Z was most likely between 33 and 130 Hz higher than it would have been if there was no start-up drift present. Table IV presents the recorded BFOs and bounds on the adjusted BFOs to remove the effects of warm-up drift. In the last column of the table, the bounds are extended taking into account the BFO noise bounds of  $[-28, +18]$  Hz established in Sec. III.

As established in Sec. III-A, the recorded BFO would be roughly 1.7 Hz lower for every 100 fpm of descent rate. As such, depending on whether the plane was tracking South or North (minimum or maximum expected BFOs, respectively, see Sec. III), bounds on the descent rate of MH370 at the times of transmission corresponding to the last 2 BFOs can be determined by subtracting the values given in the rightmost column of Table IV from the expected BFO values for level flight tracking South or North, dividing the result by 1.7 Hz and multiplying by 100 fpm. The expected BFO for a south track is approximately 260 Hz, whilst for a north track it is close to 280 Hz. Using these numbers, it is straightforward to obtain the bounds shown in Table V. Note that the bounds have been rounded to the nearest 100 fpm. Looking at all values in the table, it can be concluded that irrespective of ground track angle and for assumed ground speeds less than approximately 500 kts, under Hypothesis 1, MH370 would have been descending at between 4,900 and 15,200 fpm at 00:19:29Z, and just 8 seconds later at between 15,700 and 25,300 fpm. These descent rates are consistent with simulations of an uncontrolled phugoid descent reported in [6].

### B. Hypothesis 2: SATCOM outage due to some other reason

The descent rate bounds in the previous sub-section were derived under the hypothesis that the SATCOM outage preceding the SDU log-on event at 00:19Z was due to engine flame-out caused by insufficient fuel. Whilst this is the most likely scenario [2], it is still of interest to determine bounds on the descent rates under the alternate hypothesis that something else lead to the SDU-initiated log-on event at 00:19Z. In this case, the warm-up drift previously described would not apply. Therefore, the recorded BFOs can be treated normally (though still subject to BFO noise). In this case, the BFO bounds are as set out in Table VI. Using these bounds, the lower and upper descent rates can be calculated using the same method explained in the previous sub-section, resulting in the bounds presented in Table VII. It can be concluded from the table that even without assuming the plane ran out of fuel (as in Hypothesis 1), the recorded BFOs indicate that at 00:19:29Z the plane was descending at between 2,900 fpm and 6,800 fpm. At the time of the last SATCOM message received from MH370 (00:19:37Z), it can be concluded under Hypothesis 2 that the descent rate would have been between 13,800 fpm and 17,600 fpm.

<sup>8</sup>This was observed using 6 of the 7 log-on sequences. The log-on request BFO for log-on 7 could not be used for this due to reasons established earlier in this article.



TABLE IV  
THE LAST TWO BFOs FOR MH370 UNDER HYPOTHESIS 1

Timestamp	Recorded BFO (Hz)	BFO Range if Start-Up Drift Removed (Hz)	Extended Range Considering BFO Noise (Hz)
00:19:29Z	182	[40, 149]	[22, 177]
00:19:37Z	-2	[-132, -35]	[-150, -7]

TABLE V  
MH370 DESCENT RATES AT 00:19Z 8 MARCH 2014 UNDER HYPOTHESIS 1

Timestamp	Min. Desc. Rate, South Track	Min. Desc. Rate, North Track	Max. Desc. Rate, South Track	Max. Desc. Rate, North Track
00:19:29Z	4,900 fpm	6,100 fpm	14,000 fpm	15,200 fpm
00:19:37Z	15,700 fpm	16,900 fpm	24,100 fpm	25,300 fpm

TABLE VI  
THE LAST TWO BFOs FOR MH370 UNDER HYPOTHESIS 2

Timestamp	Recorded BFO (Hz)	BFO Bounds With Noise (Hz)
00:19:29Z	182	[164, 210]
00:19:37Z	-2	[-20, 26]

### C. Summary of Descent Rate Bounds

Combining the descent rate bounds for Hypotheses 1 and 2, outer bounds can be established for the descent rate of MH370 at 00:19:29Z and 00:19:37Z on 8 March 2014. Specifically, regardless of ground track angle and for ground speeds less than approximately 500 kts, accounting for possible BFO noise, and regardless of whether the SATCOM outage between 00:11Z and 00:19Z was due to engine flame-out or another reason, the absolute outer bounds on the possible descent rates at the times of the last two SATCOM messages from MH370 are given in Table VIII.

### D. Estimated Downwards Acceleration

When interpreting the bounds presented in Table VIII, the different conditions under which each bound was derived need to be considered. For instance the lowest rate at 00:19:29Z was derived assuming no period of SDU outage and a southwards track, whereas the highest descent rate for 00:19:37Z was derived assuming a 4-6 minute outage of the SDU with the maximum BFO decay observed in the previous 7 SDU outage events for 9M-MRO. Hence, it would not be reasonable to say that the plane could have been descending at 2,900 fpm at 00:19:29Z and 25,300 fpm 8 seconds later, which would imply a downwards acceleration on the order of 2,800 fpm per second. Whilst it is not possible to determine a precise acceleration value, a rough approximation of the average descent rate over those 8 seconds could for instance be taken using the mid-points of the bounds at each time, which would result in an average downwards acceleration of 10,500 fpm in 8 seconds or around 1,300 fpm per second, which equates to around  $6.7 \text{ ms}^{-1}$  or  $0.68g$ , where  $g$  denotes Earth's gravitational constant, which is approximately  $9.8 \text{ ms}^{-1}$ . It is straightforward to see that other reasonable methods of estimating the downward acceleration (such as taking the difference between the two minimum descent rates or the two maximum descent rates

at 00:19:29Z and 00:19:37Z) would yield similar acceleration values.

## VI. CONCLUSION

This article has discussed in detail how the BFOs logged during the MH370 accident flight related to its path towards the southern Indian Ocean. Lower and upper bounds on its descent rate at the time the last two SATCOM messages were derived<sup>9</sup>. The downwards acceleration over the 8 second interval between these two messages was found to be approximately  $0.68g$ . The derived bounds and approximate downwards acceleration rate are consistent with simulations of an uncontrolled descent near the so-called 7<sup>th</sup> arc as reported in [6]. This suggests that 9M-MRO should lie relatively close to the 7<sup>th</sup> BTO arc.

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<sup>9</sup>The conclusions regarding descent rate have been made taking into account all known factors that could have affected the last two BFOs, being BFO noise bounds, all reasonably feasible ground tracks and speeds, and possible OCXO warm-up drift. It has been implicitly assumed that there were no otherwise unknown factors that could have affected the last two BFOs.

TABLE VII  
MH370 DESCENT RATES AT 00:19Z 8 MARCH 2014 UNDER HYPOTHESIS 2

Timestamp	Min. Desc. Rate, South Track	Min. Desc. Rate, North Track	Max. Desc. Rate, South Track	Max. Desc. Rate, North Track
00:19:29Z	2,900 fpm	4,100 fpm	5,600 fpm	6,800 fpm
00:19:37Z	13,800 fpm	14,900 fpm	16,500 fpm	17,600 fpm

TABLE VIII  
RANGE OF POSSIBLE MH370 DESCENT RATES AT 00:19Z 8 MARCH 2014

Timestamp	Min. Desc. Rate	Max. Desc. Rate
00:19:29Z	2,900 fpm	15,200 fpm
00:19:37Z	13,800 fpm	25,300 fpm

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